Key Concept
The Poisson distribution is another discrete probability distribution which is important because it is often used for describing the behavior of rare events (with small probabilities).

Parameters of the Poisson Distribution
- The mean is $\mu$.
- The standard deviation is $\sigma = \sqrt{\mu}$.

Poisson Distribution
The Poisson distribution is a discrete probability distribution that applies to occurrences of some event over a specified interval. The random variable $x$ is the number of occurrences of the event in an interval. The interval can be time, distance, area, volume, or some similar unit.

Formula
$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$ where $e \approx 2.71828$

Requirements of the Poisson Distribution
- The random variable $x$ is the number of occurrences of an event over some interval.
- The occurrences must be random.
- The occurrences must be independent of each other.
- The occurrences must be uniformly distributed over the interval being used.

Parameters
- The mean is $\mu$.
- The standard deviation is $\sigma = \sqrt{\mu}$. 
Difference from a Binomial Distribution

The Poisson distribution differs from the binomial distribution in these fundamental ways:

- The binomial distribution is affected by the sample size $n$ and the probability $p$, whereas the Poisson distribution is affected only by the mean $\mu$.
- In a binomial distribution the possible values of the random variable $x$ are $0, 1, \ldots, n$, but a Poisson distribution has possible $x$ values of $0, 1, 2, \ldots$, with no upper limit.

Poisson as an Approximation to the Binomial Distribution

The Poisson distribution is sometimes used to approximate the binomial distribution when $n$ is large and $p$ is small.

**Rule of Thumb**

- $n \geq 100$
- $np \leq 10$

Poisson as an Approximation to the Binomial Distribution - $\mu$

If both of the following requirements are met,

- $n \geq 100$
- $np \leq 10$

then use the following formula to calculate $\mu$,

$\mu = np$

Recap

In this section we have discussed:

- Definition of the Poisson distribution.
- Requirements for the Poisson distribution.
- Difference between a Poisson distribution and a binomial distribution.
- Poisson approximation to the binomial.
Using a Poisson Distribution to Find Probability. In Exercises 5–8, assume that the Poisson distribution applies, and proceed to use the given mean to find the indicated probability.

244/6. If \( \mu = 0.3 \), find \( P(1) \).

Use the Poisson Distribution function in the calculator to find the answer.

Use the calculator to find the answer for the values in the formula.
Syntax: \( \text{poissonpdf}(\text{mean}, x) \)

244/8. If \( \mu = \frac{1}{6} \), find \( P(0) \).

Use the Poisson Distribution function in the calculator to find the answer.
Syntax: \( \text{poissonpdf}(\text{mean}, x) \)

244/10. Low Birth Weight

A newborn baby is considered to have a low birth weight if it weighs less than 2500 grams. Such babies often require extra care. Dutchess County, New York, has been experiencing a mean of 210.0 cases of low birth weight each year.

a. Find the mean number of low birth weight babies born each day.

b. Find the probability that on a given day, there is more than 1 baby born with a low birth weight.

"Less than or equal to one" is the complement of "greater than one."
So, \( P(x > 1) = 1 - P(x \leq 1) \)

Since \( 0.115 \leq 0.05 \), it is not unusual to have more than 1 low weight baby born on the same day.

"Less than or equal to one" is the complement of "greater than one."
So, \( P(x > 1) = 1 - P(x \leq 1) \)
244/12. Deaths from Horse Kicks A classical example of the Poisson distribution involves the number of deaths caused by horse kicks to men in the Prussian Army between 1875 and 1894. Data for 14 corps were combined for the 20-year period, and the 280 corps-years included a total of 196 deaths. After finding the mean number of deaths per corps-year, find the probability that a randomly selected corps-year has the following numbers of deaths:

a. 0
b. 1
c. 2
d. 3
e. 4

The actual results consisted of these frequencies: 0 deaths (in 144 corps-years); 1 death (in 91 corps-years); 2 deaths (in 32 corps-years); 3 deaths (in 11 corps-years); 4 deaths (in 2 corps-years). Compare the actual results to those expected by using the Poisson probabilities. Does the Poisson distribution serve as a good device for predicting the actual results?

Use the TI calculator to generate the probabilities as shown in the screens on the right.

Since the actual values (f) are close to the Expected values, the Poisson Distribution is a good device for predicting the actual results.

The Excel program used here is available through the Technology section of the Important Links webpage or directly at http://cfcc.edu/faculty/cmoore/0403-PoissonDist.xls

You may use Statdisk software to generate the Poisson Distribution values as shown below.
245/14. Disease Cluster
Neuroblastoma, a rare form of malignant tumor, occurs in 11 children in a million, so its probability is 0.000011. Four cases of neuroblastoma occurred in Oak Park, Illinois, which had 12,429 children.

a. Assuming that neuroblastoma occurs as usual, find the mean number of cases in groups of 12,429 children.
b. Find the probability that the number of neuroblastoma cases in a group of 12,429 children is 0 or 1.
c. What is the probability of more than one case of neuroblastoma?
d. Does the cluster of four cases appear to be attributable to random chance? Why or why not?

245/16. Life Insurance
There is a 0.9986 probability that a randomly selected 30-year-old male lives through the year (based on data from the U.S. Department of Health and Human Services). A Fidelity life insurance company charges $161 for insuring that the male will live through the year. If the male does not survive the year, the policy pays out $100,000 as a death benefit. Assume that the company sells 1300 such policies to 30-year-old males, so it collects $209,300 in policy payments. The company will make a profit if the number of deaths in this group is two or fewer.

a. What is the mean number of deaths in such groups of 1300 males?
b. Use the Poisson distribution to find the probability that the company makes a profit from the 1300 policies.
c. Use the binomial distribution to find the probability that the company makes a profit from the 1300 policies, then compare the result to the result found in part (b).

to four-decimal places, the two answers are the same: 0.7253. We see that both \( n \geq 100 \) and \( np \leq 10 \) are true because \( n = 1300 \) and \( p = 0.0014 \) to yield \( np = 1300(0.0014) = 1.82 \). Thus, the Poisson probability may be used to estimate the Binomial probability.