This section presents methods for testing a claim about a population mean, given that the population standard deviation is a known value. This section uses the normal distribution with the same components of hypothesis tests that were introduced in Section 8-2.

### Key Concept

### Notation

- \( n \): sample size
- \( \bar{x} \): sample mean
- \( \mu_x \): population mean of all sample means from samples of size \( n \)
- \( \sigma \): known value of the population standard deviation

### Requirements for Testing Claims About a Population Mean (with \( \sigma \) Known)

1. The sample is a simple random sample.
2. The value of the population standard deviation \( \sigma \) is known.
3. Either or both of these conditions is satisfied: The population is normally distributed or \( n > 30 \).

### Test Statistic for Testing a Claim About a Mean (with \( \sigma \) Known)

\[
Z = \frac{\bar{x} - \mu_x}{\sigma / \sqrt{n}}
\]
Example:
People have died in boat accidents because an obsolete estimate of the mean weight of men was used. Using the weights of the simple random sample of men from Data Set 1 in Appendix B, we obtain these sample statistics: $n = 40$ and $\bar{x} = 172.55$ lb. Research from several other sources suggests that the population of weights of men has a standard deviation given by $\sigma = 26$ lb. Use these results to test the claim that men have a mean weight greater than 166.3 lb, which was the weight in the National Transportation and Safety Board’s recommendation M-04-04. Use a 0.05 significance level, and use the $P$-value method outlined in Figure 8-8.

Example:
Requirements are satisfied: simple random sample, $\sigma$ is known (26 lb), sample size is 40 ($n > 30$)

Step 1: Express claim as $\mu > 166.3$ lb
Step 2: alternative to claim is $\mu \leq 166.3$ lb
Step 3: $\mu > 166.3$ lb does not contain equality, it is the alternative hypothesis:

$H_0$: $\mu = 166.3$ lb null hypothesis
$H_1$: $\mu > 166.3$ lb alternative hypothesis and original claim

Example:
Step 4: significance level is $\alpha = 0.05$
Step 5: claim is about the population mean, so the relevant statistic is the sample mean (172.55 lb), $s$ is known (26 lb), sample size greater than 30
Step 6: calculate $z$

$$z = \frac{\bar{x} - \mu}{\sigma} \div \sqrt{n} = \frac{172.55 - 166.3}{26} \div \sqrt{40} = 1.52$$

right-tailed test, so $P$-value is the area to the right of $z = 1.52$;

Example:
Table A-2: area to the left of $z = 1.52$ is 0.9357, so the area to the right is $1 - 0.9357 = 0.0643$. The $P$-value is 0.0643
Step 7: The $P$-value of 0.0643 is greater than the significance level of $\alpha = 0.05$, we fail to reject the null hypothesis.
Example:
The $P$-value of 0.0643 tells us that if men have a mean weight given by $\mu = 166.3$ lb, there is a good chance (0.0643) of getting a sample mean of 172.55 lb. A sample mean such as 172.55 lb could easily occur by chance. There is not sufficient evidence to support a conclusion that the population mean is greater than 166.3 lb, as in the National Transportation and Safety Board’s recommendation.

Example:
The traditional method: Use $z = 1.645$ instead of finding the $P$-value. Since $z = 1.52$ does not fall in the critical region, again fail to reject the null hypothesis.

Confidence Interval method: Use a one-tailed test with $a = 0.05$, so construct a 90% confidence interval: $165.8 < m < 179.3$

The confidence interval contains 166.3 lb, we cannot support a claim that $m$ is greater than 166.3. Again, fail to reject the null hypothesis.

Underlying Rationale of Hypothesis Testing
If, under a given assumption, there is an extremely small probability of getting sample results at least as extreme as the results that were obtained, we conclude that the assumption is probably not correct.

When testing a claim, we make an assumption (null hypothesis) of equality. We then compare the assumption and the sample results and we form one of the following conclusions:

Underlying Rationale of Hypotheses Testing - cont
• If the sample results (or more extreme results) can easily occur when the assumption (null hypothesis) is true, we attribute the relatively small discrepancy between the assumption and the sample results to chance.
• If the sample results cannot easily occur when that assumption (null hypothesis) is true, we explain the relatively large discrepancy between the assumption and the sample results by concluding that the assumption is not true, so we reject the assumption.
Recap

In this section we have discussed:
• Requirements for testing claims about population means, σ known.
• P-value method.
• Traditional method.
• Confidence interval method.
• Rationale for hypothesis testing.

436/6. Weights of Pennies The U.S. Mint has a specification that pennies have a mean weight of 2.5 g. Assume that weights of pennies have a standard deviation of 0.0165 g and use the accompanying Minitab display to test the claim that the sample is from a population with a mean that is less than 2.5 g. These weights were obtained using the 47 weights of post-1983 pennies listed in Data Set 20 in Appendix B.

436/8. Red Blood Cell Count A simple random sample of 50 adults is obtained, and each person’s red blood cell count (in cells per microliter) is measured. The sample mean is 5.23. The population standard deviation for red blood cell counts is 0.54. Use a 0.01 significance level to test the claim that the sample is from a population with a mean less than 5.4, which is a value often used for the upper limit of the range of normal values. What do the results suggest about the sample group?

437/12. Loaded Die When a fair die is rolled many times, the outcomes of 1, 2, 3, 4, 5, and 6 are equally likely, so the mean of the outcomes should be 3.5. The author drilled holes into a die and loaded it by inserting lead weights, then rolled it 16 times to obtain a mean of 2.9375. Assume that the standard deviation of the outcomes is 1.7078, which is the standard deviation for a fair die. Use a 0.05 significance level to test the claim that outcomes from the loaded die have a mean different from the value of 3.5 expected with a fair die. Is there anything about the sample data suggesting that the methods of this section should not be used?
Weights of Bears
The health of the bear population in Yellowstone National Park is monitored by periodic measurements taken from anesthetized bears. A sample of 54 bears has a mean weight of 182.9 lb. Assuming that \( \sigma \) is known to be 121.8 lb, use a 0.05 significance level to test the claim that the population mean of all such bear weights is greater than 150 lb.

Sample data support the claim that the population mean of all such bear weights is greater than 150 lb.

FICO Credit Scores
A simple random sample of FICO credit rating scores is obtained, and the scores are listed below. As of this writing, the mean FICO score was reported to be 678. Assuming the standard deviation of all FICO scores is known to be 58.3, use a 0.05 significance level to test the claim that these sample FICO scores come from a population with a mean equal to 678.

Enter data into TI calculator and perform the z test as shown on the screens.

Since \( P = 8.37 \times 10^{-7} \leq 0.05 \), reject \( H_0 \).

Box 1: Reject the claim.

Conclusion: There is sufficient evidence to warrant rejection of the claim that these sample FICO scores come from a population with a mean equal to 678.

California Speeding
Listed below are recorded speeds (in mph) of randomly selected cars traveling on a section of Highway 405 in Los Angeles (based on data from Sigalert). That part of the highway has a posted speed limit of 65 mph. Assume that the standard deviation \( \sigma \) of speeds is 5.7 mph and use a 0.01 significance level to test the claim that the sample is from a population with a mean that is greater than 65 mph.

Sample data support the claim that the sample is from a population with a mean that is greater than 65 mph.