Chapter 4 Probability

4-1 Review and Preview
4-2 Basic Concepts of Probability
4-3 Addition Rule
4-4 Multiplication Rule: Basics
4-7 Counting

To find more information about statistics and probability, visit Rice Virtual Lab in Statistics at http://davidmlane.com/hyperstat/probability.html

The following lesson is in Course Documents of CourseCompass.

155S4.1-2_3 Review & Preview; Basic Concepts of Probability

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**Key Concept**

This section presents three approaches to finding the probability of an event.

The most important objective of this section is to learn how to interpret probability values.

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**Review**

Necessity of sound sampling methods.

Common measures of characteristics of data Mean & Standard deviation

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**Preview**

Rare Event Rule for Inferential Statistics:

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

Statisticians use the rare event rule for inferential statistics.

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**Events and Sample Space**

- **Event**
  - any collection of results or outcomes of a procedure

- **Simple Event**
  - an outcome or an event that cannot be further broken down into simpler components

- **Sample Space**
  - for a procedure consists of all possible simple events; that is, the sample space consists of all outcomes that cannot be broken down any further

\[ S = \{ H, T \} \]
**Notation for Probabilities**

- $P$ - denotes a probability.
- $A$, $B$, and $C$ - denote specific events.
- $P(A)$ - denotes the probability of event $A$ occurring.

**Basic Rules for Computing Probability**

**Rule 1: Relative Frequency Approximation of Probability**

Conduct (or observe) a procedure, and count the number of times event $A$ actually occurs. Based on these actual results, $P(A)$ is approximated as follows:

\[
P(A) = \frac{\text{# of times } A \text{ occurred}}{\text{# of times procedure was repeated}}
\]

**Law of Large Numbers**

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

**Rule 2: Classical Approach to Probability**

(Requires Equally Likely Outcomes)

Assume that a given procedure has $n$ different simple events and that each of those simple events has an equal chance of occurring. If event $A$ can occur in $s$ of these $n$ ways, then

\[
P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}
\]

**Rule 3: Subjective Probabilities**

$P(A)$, the probability of event $A$, is estimated by using knowledge of the relevant circumstances.
Probability Limits

Always express a probability as a fraction or decimal number between 0 and 1.

- The probability of an impossible event is 0.
- The probability of an event that is certain to occur is 1.
- For any event A, the probability of A is between 0 and 1 inclusive.
  That is, $0 \leq P(A) \leq 1$.

Possible Values for Probabilities

Complementary Events

The complement of event $A$, denoted by $\overline{A}$, consists of all outcomes in which the event $A$ does not occur.

Rounding Off Probabilities

When expressing the value of a probability, either give the exact fraction or decimal or round off final decimal results to three significant digits. (Suggestion: When a probability is not a simple fraction such as 2/3 or 5/9, express it as a decimal so that the number can be better understood.)

Here are some rounding examples; each number is rounded to four, three, and two significant digits. http://www.purplemath.com/modules/rounding2.htm

1. Round 742,396
   - 742,400 (four significant digits)
   - 742,000 (three significant digits)
   - 740,000 (two significant digits)

2. Round 0.07284
   - 0.07284 (four significant digits)
   - 0.0728 (three significant digits)
   - 0.073 (two significant digits)

3. Round 231.45
   - 231.5 (four significant digits)
   - 231 (three significant digits)
   - 230 (two significant digits)
Odds

The actual odds against event A occurring are the ratio \( P(A)/P(\bar{A}) \), usually expressed in the form of \( a:b \) (or “\( a \) to \( b \)”), where \( a \) and \( b \) are integers having no common factors.

The actual odds in favor of event A occurring are the ratio \( P(A)/P(\bar{A}) \), which is the reciprocal of the actual odds against the event. If the odds against A are \( a:b \), then the odds in favor of A are \( b:a \).

The payoff odds against event A occurring are the ratio of the net profit (if you win) to the amount bet.

\[
\text{payoff odds against event } A = \text{(net profit)} : \text{(amount bet)}
\]

Recap

In this section we have discussed:

- Rare event rule for inferential statistics.
- Probability rules.
- Law of large numbers.
- Complementary events.
- Rounding off probabilities.
- Odds.

In Exercises 5–12, express the indicated degree of likelihood as a probability value between 0 and 1.

154/5. Weather A WeatherBug forecast for the author’s home was stated as: “Chance of rain: 80%.”

\[
P(\text{rain}) = \frac{20}{100} = \frac{0.2}{1} = 0.2
\]

154/6. Births When a baby is born, there is approximately a 50-50 chance that the baby is a girl.

\[
P(\text{girl}) = \frac{1}{2} = 0.5
\]

154/8. Roulette When playing roulette in the Mirage Casino, you have 18 chances out of 38 of winning if you bet that the outcome is an odd number.

\[
P(\text{win}) = \frac{18}{38} = \frac{9}{19} = 0.474
\]

154/10. Days When randomly selecting a day of the week, you are certain to select a day containing the letter y.

\[
P(\text{y in name of day}) = 1
\]
**Genotypes** In Example 4, we noted that a study involved equally likely genotypes represented as AA, Aa, aA, and aa. If one of these genotypes is randomly selected as in Example 4, what is the probability that the outcome is AA? Is obtaining AA unusual?

\[
P(\text{AA}) = \frac{1}{4} = 0.25
\]

If \( P(\text{Event}) \leq 0.05 \), then the event is unusual.

**Polygraph Test** Refer to the sample data in Table 4-1.

a. How many responses were actually lies?

b. If one of the responses is randomly selected, what is the probability that it is a lie? (Express the answer as a fraction.)

**Mendelian Genetics** When Mendel conducted his famous genetics experiments with peas, one sample of offspring consisted of 428 green peas and 152 yellow peas. Based on those results, estimate the probability of getting an offspring pea that is green. Is the result reasonably close to the expected value of \( \frac{3}{4} \), as claimed by Mendel?

\[
P(\text{green}) = \frac{\# \text{ of green}}{\text{total available}} = \frac{428}{428 + 152} = \frac{428}{580} = 0.738
\]

\[
\frac{3}{4} = 0.750
\]

Reasonably close.

**Gender Selection** In updated results from a test of MicroSort's XSORT gender-selection technique, 726 births consisted of 668 baby girls and 58 baby boys (based on data from the Genetics & IVF Institute). Based on these results, what is the probability of a girl born to a couple using MicroSort's XSORT method? Does it appear that the technique is effective in increasing the likelihood that a baby will be a girl?

\[
P(\text{girl}) = \frac{668}{726} = 0.920
\]

\[
\frac{668}{726} = 0.920
\]

\[
\frac{668 + 58}{726} = 0.920
\]
Consider an event to be "unusual" if its probability is less than or equal to 0.05.

**156/28. Bumped from a Flight** Among 15,378 Delta airline passengers randomly selected, 3 were bumped from a flight against their wishes (based on data from the U. S. Department of Transportation). Find the probability that a randomly selected passenger is involuntarily bumped. Is such bumping unusual? Does such bumping pose a serious problem for Delta passengers in general? Why or why not?

\[
P(\text{involuntarily bumped}) = \frac{3}{15378} \\
= 0.000195 \\n\approx 0.0002
\]

**156/32. Personal Calls at Work** USA Today reported on a survey of office workers who were asked how much time they spend on personal phone calls per day. Among the responses, 1065 reported times between 1 and 10 minutes, 240 reported times between 11 and 30 minutes, 14 reported times between 31 and 60 minutes, and 66 said that they do not make personal calls. If a worker is randomly selected, what is the probability the worker does not make personal calls. Is it unusual for a worker to make no personal calls?

\[
P(\text{not make personal calls}) = \frac{66}{1065 + 240 + 14 + 66} \\
= \frac{66}{1385} \\
\approx 0.0477
\]

**156/33. Gender of Children: Constructing Sample Space** This section included a table summarizing the gender outcomes for a couple planning to have three children.

- a. Construct a similar table for a couple planning to have two children.
- b. Assuming that the outcomes listed in part (a) are equally likely, find the probability of getting two girls.
- c. Find the probability of getting exactly one child of each gender.