Chapter 4 Probability

4-1 Review and Preview
4-2 Basic Concepts of Probability
4-3 Addition Rule
4-4 Multiplication Rule: Basics
4-7 Counting

The following lesson is in Course Documents of CourseCompass.
S2.3D.MAT 155 Chapter 4 - Probability
This contains some notes for Chapter 4 Probability.
It contains the following topics relative to probability: Fundamentals; Addition Rule; Multiplication Rule: Basics; Multiplication Rule: Complements and Conditional Probability; Probabilities through Simulations; Counting

Key Concept

The basic multiplication rule is used for finding \( P(A \text{ and } B) \), the probability that event \( A \) occurs in a first trial and event \( B \) occurs in a second trial. If the outcome of the first event \( A \) somehow affects the probability of the second event \( B \), it is important to adjust the probability of \( B \) to reflect the occurrence of event \( A \).

Notation

\[
P(A \text{ and } B) = P(\text{event } A \text{ occurs in a first trial and event } B \text{ occurs in a second trial})
\]

Tree Diagrams

A tree diagram is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are sometimes helpful in determining the number of possible outcomes in a sample space, if the number of possibilities is not too large.

This figure summarizes the possible outcomes for a true/false question followed by a multiple choice question.
Conditional Probability Key Point

We must adjust the probability of the second event to reflect the outcome of the first event.

Conditional Probability Important Principle

The probability for the second event $B$ should take into account the fact that the first event $A$ has already occurred.

Notation for Conditional Probability

$P(B|A)$ represents the probability of event $B$ occurring after it is assumed that event $A$ has already occurred (read $B|A$ as “$B$ given $A$.”)

Dependent Events

Two events are dependent if the occurrence of one of them affects the probability of the occurrence of the other, but this does not necessarily mean that one of the events is a cause of the other.

Dependent and Independent

Two events $A$ and $B$ are independent if the occurrence of one does not affect the probability of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of the occurrence of the others.)

If $A$ and $B$ are not independent, they are said to be dependent.

Formal Multiplication Rule

- $P(A \text{ and } B) = P(A) \cdot P(B|A)$
- Note that if $A$ and $B$ are independent events, $P(B|A)$ is really the same as $P(B)$.

Intuitive Multiplication Rule

When finding the probability that event $A$ occurs in one trial and event $B$ occurs in the next trial, multiply the probability of event $A$ by the probability of event $B$, but be sure that the probability of event $B$ takes into account the previous occurrence of event $A$. 
Applying the Multiplication Rule

Caution
When applying the multiplication rule, always consider whether the events are independent or dependent, and adjust the calculations accordingly.

Multiplication Rule for Several Events
In general, the probability of any sequence of independent events is simply the product of their corresponding probabilities.

Treating Dependent Events as Independent
Some calculations are cumbersome, but they can be made manageable by using the common practice of treating events as independent when small samples are drawn from large populations. In such cases, it is rare to select the same item twice.

The 5% Guideline
If \( n < 0.05N \), treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent).

Principle of Redundancy
One design feature contributing to reliability is the use of redundancy, whereby critical components are duplicated so that if one fails, the other will work. For example, single-engine aircraft now have two independent electrical systems so that if one electrical system fails, the other can continue to work so that the engine does not fail.
Summary of Fundamentals

• In the **addition rule**, the word “or” in \( P(A \text{ or } B) \) suggests addition. Add \( P(A) \) and \( P(B) \), being careful to add in such a way that every outcome is counted only once.

• In the **multiplication rule**, the word “and” in \( P(A \text{ and } B) \) suggests multiplication. Multiply \( P(A) \) and \( P(B) \), but be sure that the probability of event \( B \) takes into account the previous occurrence of event \( A \).

Recap

In this section we have discussed:

- Notation for \( P(A \text{ and } B) \).
- Tree diagrams.
- Notation for conditional probability.
- Independent events.
- Formal and intuitive multiplication rules.

In Exercises 5–12, for each given pair of events, classify the two events as independent or dependent.

174/6. Finding that your car radio works.
Finding that your car headlights work.

174/10. Finding that your calculator works.
Finding that your computer works.
In Exercises 13–16, use the sample data in Table 4-1. (See Example 1.)

**Polygraph Test** If 3 of the 98 test subjects are randomly selected without replacement, find the probability that they all had false positive results. Is it unusual to randomly select 3 subjects without replacement and get 3 results that are all false positive results? Explain.

<table>
<thead>
<tr>
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175/21. **Guessing** A quick quiz consists of a true/false question followed by a multiple-choice question with four possible answers (a, b, c, d). An unprepared student makes random guesses for both answers.

a. Consider the event of being correct with the first guess and the event of being correct with the second guess. Are those two events independent?

b. What is the probability that both answers are correct?

c. Based on the results, does guessing appear to be a good strategy?

\[ P(C \text{ and } C) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} = 0.125 \]

(c) Since 0.125 $< 0.05$; it is not unusual to guess 2 correct.

175/22. **Acceptance Sampling** With one method of a procedure called acceptance sampling, a sample of items is randomly selected without replacement and the entire batch is accepted if every item in the sample is okay. The Telektronics Company manufactured a batch of 400 backup power supply units for computers, and 8 of them are defective. If 3 of the units are randomly selected for testing, what is the probability that the entire batch will be accepted?

\[ P(\text{6 good}) = \frac{\binom{322}{3}}{\binom{400}{3}} = \frac{322 \cdot 321 \cdot 320}{400 \cdot 399 \cdot 398} = 0.941 \]

175/28. **Redundancy** The FAA requires that commercial aircraft used for flying in instrument conditions must have two independent radios instead of one. Assume that for a typical flight, the probability of a radio failure is 0.002. What is the probability that a particular flight will be threatened with the failure of both radios? Describe how the second independent radio increases safety in this case.

\[ P(F \text{ and } F) = (0.002)(0.002) = 0.000004 \]

\[ P(F) = 0.002 \approx \frac{2}{250,000} \]

Since $\frac{2}{250,000}$ is extremely small, we are extremely likely to have a good (functioning) radio in 2/250,000.

175/30. **Car Ignition Systems** A quality control analyst randomly selects 3 different car ignition systems from a manufacturing process that has just produced 200 systems, including 5 that are defective.

a. Does this selection process involve independent events?

b. What is the probability that all 3 ignition systems are good? (Do not treat the events as independent.)

c. Use the 5% guideline for treating the events as independent, and find the probability that all 3 ignition systems are good.

d. Which answer is better: The answer from part (b) or the answer from part (c)? Why?

(a) not independent

(b) \[ P(G) = \frac{195}{200} \]

(c) \[ P(GGG) = \frac{195 \cdot 194 \cdot 193}{200 \cdot 199 \cdot 198} \approx 0.979 \]

(d) Answer from (a), technically correct.