Equations and Solutions

An equation is a statement that two expressions are equal. To solve an equation in one variable is to find all the values of the variable that make the equation true. Each of these numbers is a solution of the equation. The set of all solutions of an equation is its solution set.

A linear equation in one variable is an equation that can be expressed in the form $mx + b = 0$, where $m$ and $b$ are real numbers and $m \neq 0$.

Equations that have the same solution set are equivalent equations.

Equation-Solving Principles

**The Addition Principle:**
If $a = b$ is true, then $a + c = b + c$ is true.

**The Multiplication Principle:**
If $a = b$ is true, then $ac = bc$ is true.

Distance Formula
The distance $d$ traveled by an object moving at rate $r$ in time $t$ is given by

\[ d = rt. \]

Simple Interest
\[ I = Prt. \]

- $I$ = the simple interest ($\)$
- $P$ = the principal ($\)$
- $r$ = the interest rate ($\%$)
- $t$ = time (years)

Zeros of Linear Functions
An input $c$ of a function $f$ is called a zero of the function, if the output for the function is 0 when the input is $c$. That is, $c$ is a zero of $f$ if $f(c) = 0$.

\[ f(c) = 0 \rightarrow x = c \text{ and } y = 0 \]

A linear function $f(x) = mx + b$, with $m \neq 0$, has exactly one zero.
1.5 Linear Equations, Functions, Zeros and Applications

1.5.1 Solve \((\frac{6}{5})y + 3 = \frac{3}{10}\)

1.5.10 Solve \(10x - 3 = 8 + 10x\)

1.5.22 Solve \(4x + 3 = 2x - 7\)

1.5.30 Solve \(4(3x + 2) - 7 = 3(x - 2)\)

The answer is \(x = -5\).

1.5.34 Oil Consumption

Projections indicate that the global demand for oil will be 103 million barrels per day in 2015. This is about a 23% increase over the daily demand in 2005. (Source: U.S. Energy Information Administration) What was the daily global demand for oil in 2005?

This is incorrect procedure and answer.

1.5.38 Where the Textbook Dollar Goes

Of each dollar spent on textbooks at college bookstores, 23.2 cents goes to the college store for profit, store operations, and personnel. On average, a college student at a four-year college spends $940 per year for textbooks. (Source: College Board) How much of this expenditure goes to the college store?

1.5.40 Nutrition

A slice of carrot cake from the popular restaurant The Cheesecake Factory contains 1560 calories. This is three-fourths of the average daily calorie requirement for many adults. (Source: The Center for Science in the Public Interest) Find the average daily calorie requirement for these adults.

We failed to work this problem in class.

Let \(x = \) $ amount of sales per month

Choice 1: monthly salary = $1800
Choice 2: monthly salary = $1600 + 0.04x

Choice 1 = Choice 2 yields the equation, \(1800 = 1600 + 0.04x\).

Solve this equation to get \(x = \) $5000 of sales per month would yield the same monthly salary for Julia.
### Garden Dimensions
The children at Tiny Tots Day Care plant a rectangular vegetable garden with a perimeter of 39 m. The length is twice the width. Find the dimensions of the garden.

**Graph, calculate perimeter and area of rectangle given the width and the relationship between length and width**

Use this to graph, calculate perimeter and area of rectangle given the width and the relationship between length and width.

http://cfcc.edu/faculty/cmoore/RectangleL_aW.html

### Water Weight
Water accounts for 60% of a man’s weight (Source: National Institute for Fitness and Sport). Emilio weighs 186 lb. How much of his body weight is water?

### Traveling Downstream
Angelos kayak travels 14 km/hr in still water. If the rivers current flows at a rate of 2 km/hr, how long will it take him to travel 20 km downstream?

**So, downstream speed of kayak is 14 + 2 = 16 km/hr.**

\[
d = rt\] yields \(20 = 16t\) where \(t\) = time (hours) to travel downstream.

16t = 20 gives 6 = 20/16 = 5/4 or 1.25 hours.

### Student Loans
Dimitri’s two student loans total $9000. One loan is at 5% simple interest and the other is at 6% simple interest. At the end of 1 yr, Dimitri owes $492 in interest. What is the amount of each loan?

### Finding the Zero
Find the zero of the linear function: \(f(x) = 8x + 2\).

**Find the zero of the linear function: \(f(x) = \frac{2}{5}x - 10\).**
In Exercises 87-92, use the given graph to find each of the following: (a) the x-intercept and (b) the zero of the function.

(a) x-intercept (-2,0)  
(b) zero: -2 in x = -2

\[ y = 0 \rightarrow x = -2 \]

1.5 Linear Equations, Functions, Zeros and Applications

148/92.

148/98. Solve \( A = \frac{1}{2} h (b_1 + b_2) \), for \( h \).

\[
\begin{align*}
A &= \frac{1}{2} h (b_1 + b_2) \\
2A &= h (b_1 + b_2) \\
h &= \frac{2A}{b_1 + b_2} \\
\Rightarrow h &= 2A / (b_1 + b_2) \quad \text{must use ( )}
\end{align*}
\]

148/106. Solve \( 3x + 4y = 12 \), for \( y \).

\[ y = -\frac{3}{4} x + 3 \]

148/111. Solve \( z = xy - xy^2 \), for \( x \).

\[
\begin{align*}
\frac{z}{y^2} &= x - y \quad \text{(Wrong)} \\
x &= \frac{z}{y^2} + y \\
x &= \frac{z + y^2}{y^2} \\
x &= \frac{z + y^2}{y^2} \quad \text{must use ( )}
\end{align*}
\]

148/119. Solve \( 200 - x \rightarrow 80 + x \)

\[
\begin{align*}
\$0.0699/\text{lb} & \rightarrow \$0.0719/\text{lb} \\
80 & \rightarrow 6.99 \\
6.99 & \rightarrow 6.99 \text{cents} \\
\frac{6.99 \text{ cents}}{100 \text{ lb}} & = 0.0699/\text{lb}
\end{align*}
\]

\[
\begin{align*}
\text{Price} \times \text{lb} &= \text{Cost} \\
\text{Cost} &= 0.0719 - 0.0699 \\
\text{Price} & = 0.0020 \\
\% \text{ of} 0.0699 &= 0.0020 \\
0.0286 &= 2.9\% \text{ inc.} \quad \text{per lb}
\end{align*}
\]