The Standard Normal Distribution

Key Concept

This section presents the **standard normal distribution** which has three properties:

- Its graph is bell-shaped.
- Its mean is equal to 0 ($\mu = 0$).
- Its standard deviation is equal to 1 ($\sigma = 1$).

Develop the skill to find areas (or probabilities or relative frequencies) corresponding to various regions under the graph of the standard normal distribution. Find z-scores that correspond to area under the graph.

Uniform Distribution

A continuous random variable has a **uniform distribution** if its values are spread evenly over the range of probabilities. The graph of a uniform distribution results in a rectangular shape.

A density curve is the graph of a continuous probability distribution. It must satisfy the following properties:

1. The total area under the curve must equal 1.
2. Every point on the curve must have a vertical height that is 0 or greater. (That is, the curve cannot fall below the x-axis.)
Area and Probability

Because the total area under the density curve is equal to 1, there is a correspondence between area and probability.

In Course Documents of CourseCompass, see S3.D2.MAT 155 Using Technology for Chapter 6 155Ch6 Technology (Package file)
This lesson illustrates how to use technology (TI-83/84 calculator, Statdisk, and Excel) to solve problems from Chapter 6 Normal Probability Distribution.

http://cfcc.edu/mathlab/geogebra/normal_curve_aba.html
http://cfcc.edu/mathlab/geogebra/uniform.html

Using Area to Find Probability

Given the uniform distribution illustrated, find the probability that a randomly selected voltage level is greater than 124.5 volts.

Shaded area represents voltage levels greater than 124.5 volts.
Correspondence between area and probability: 0.25.

Standard Normal Distribution

The standard normal distribution is a normal probability distribution with \( m = 0 \) and \( s = 1 \).
The total area under its density curve is equal to 1.

Finding Probabilities – Other Methods

- STATDISK
- Minitab
- Excel
- TI-83/84 Plus
- GeoGebra
Methods for Finding Normal Distribution Areas

Table A-2
Standard Normal (Z) Distribution: Cumulative Area from the Left

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<tr>
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<th>.00</th>
<th>.01</th>
<th>.02</th>
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</tbody>
</table>

Using Table A-2

1. It is designed only for the standard normal distribution, which has a mean of 0 and a standard deviation of 1.
2. It is on two pages, with one page for negative z-scores and the other page for positive z-scores.
3. Each value in the body of the table is a cumulative area from the left up to a vertical boundary above a specific z-score.
Using Table A-2

4. When working with a graph, avoid confusion between z-scores and areas.

**z Score**

- **Distance** along horizontal scale of the standard normal distribution; refer to the leftmost column and top row of Table A-2.

**Area**

- **Region** under the curve; refer to the values in the body of Table A-2.

5. The part of the z-score denoting hundredths is found across the top.

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Example - Thermometers

The Precision Scientific Instrument Company manufactures thermometers that are supposed to give readings of 0°C at the freezing point of water. Tests on a large sample of these instruments reveal that at the freezing point of water, some thermometers give readings below 0°C (denoted by negative numbers) and some give readings above 0°C (denoted by positive numbers). Assume that the mean reading is 0°C and the standard deviation of the readings is 1.00°C. Also assume that the readings are normally distributed. If one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.27°C.

**Example - cont**

Look at Table A-2

<table>
<thead>
<tr>
<th>z</th>
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</table>

The probability of randomly selecting a thermometer with a reading less than 1.27°C is 0.8980.

Or 89.80% will have readings below 1.27°C.
Example - Thermometers Again

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.

Probability of randomly selecting a thermometer with a reading above –1.23° is 0.8907.

89.07% of the thermometers have readings above –1.23 degrees.

Example - Thermometers III

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.

\[
P(z < -2.00) = 0.0228
\]
\[
P(z < 1.50) = 0.9332
\]
\[
P(-2.00 < z < 1.50) = 0.9332 - 0.0228 = 0.9104
\]
The probability that the chosen thermometer has a reading between –2.00 and 1.50 degrees is 0.9104.

If many thermometers are selected and tested at the freezing point of water, then 91.04% of them will read between –2.00 and 1.50 degrees.

Notation

\[P(a < z < b) = \text{probability } z \text{ score is between } a \text{ and } b.\]
\[P(z > a) = \text{probability } z \text{ score is greater than } a.\]
\[P(z < a) = \text{probability } z \text{ score is less than } a.\]

Finding a z Score When Given a Probability Using Table A-2

1. Draw a bell-shaped curve and identify the region under the curve that corresponds to the given probability. If that region is not a cumulative region from the left, work instead with a known region that is a cumulative region from the left.
2. Using the cumulative area from the left, locate the closest probability in the body of Table A-2 and identify the corresponding \(z\) score.

Finding z Scores When Given Probabilities

(z score will be positive)

Finding the 95th Percentile
Recap

In this section we have discussed:

- Density curves.
- Relationship between area and probability.
- Standard normal distribution.
- Using Table A-2.

Finding $z$ Scores When Given Probabilities - cont

(One $z$ score will be negative and the other positive)

Finding the Bottom 2.5% and Upper 2.5%

Continuous Uniform Distribution. In Exercises 5–8, refer to the continuous uniform distribution depicted in Figure 6-2. Assume that a voltage level between 123.0 volts and 125.0 volts is randomly selected, and find the probability that the given voltage level is selected.

- Less than 123.5 volts
- Between 124.1 volts and 124.5 volts

Uniform Distribution - http://cfcc.edu/mathlab/geogebra/uniform.html

Continuous Uniform Distribution. In Exercises 5–8, refer to the continuous uniform distribution depicted in Figure 6-2. Assume that a voltage level between 123.0 volts and 125.0 volts is randomly selected, and find the probability that the given voltage level is selected.

- Less than 123.5 volts
- Between 124.1 volts and 124.5 volts

Uniform Distribution - http://cfcc.edu/mathlab/geogebra/uniform.html
The Standard Normal Distribution. In Exercises 9–12, find the area of the shaded region. The graph depicts the standard normal distribution with mean 0 and standard deviation 1.

\[
P(X < -0.75) = 0.22663 \\
P(X > -0.75) = 0.77337
\]

Normal curve left of x or right of x - [link]

Normal Distribution \( a \leq x \leq b \) - [link]

The Standard Normal Distribution. In Exercises 13–16, find the indicated z score. The graph depicts the standard normal distribution with mean 0 and standard deviation 1.

\[
\text{invNorm}(-.9418) \\
-1.570064956
\]

Normal curve left of x or right of x - [link]
Standard Normal Distribution. In Exercises 17–36, assume that thermometer readings are normally distributed with a mean of 0° C and a standard deviation of 1.00° C. A thermometer is randomly selected and tested. In each case, draw a sketch, and find the probability of each reading. (The given values are in Celsius degrees.) If using technology instead of Table A-2, round answers to four decimal places.

268/18 Less than -2.75
268/22 Greater than 2.33

268/26 Between 1.00 and 3.00

Standard Normal Distribution. In Exercises 37–40, find the indicated area under the curve of the standard normal distribution, then convert it to a percentage and fill in the blank. The results form the basis for the range rule of thumb and the empirical rule introduced in Section 3-3.

268/37 About ____% of the area is between \( z = -1 \) and \( z = 1 \) (or within 1 standard deviation of the mean).

268/38 About ____% of the area is between \( z = -2 \) and \( z = 2 \) (or within 2 standard deviation of the mean).
Basis for the Range Rule of Thumb and the Empirical Rule. In Exercises 37–40, find the indicated area under the curve of the standard normal distribution, then convert it to a percentage and fill in the blank. The results form the basis for the range rule of thumb and the empirical rule introduced in Section 3-3.

268/39. About __% of the area is between $z = -3$ and $z = 3$ (or within 3 standard deviations of the mean).

Finding Critical Values. In Exercises 41–44, find the indicated value.

269/42. $z_{0.01}$

Basis for the Range Rule of Thumb and the Empirical Rule. In Exercises 37–40, find the indicated area under the curve of the standard normal distribution, then convert it to a percentage and fill in the blank. The results form the basis for the range rule of thumb and the empirical rule introduced in Section 3-3.

268/40. About __% of the area is between $z = -3.5$ and $z = 3.5$ (or within 3.5 standard deviations of the mean).

Finding Critical Values. In Exercises 41–44, find the indicated value.

269/44. $z_{0.02}$
Finding Probability. In Exercises 45–48, assume that the readings on the thermometers are normally distributed with a mean of 0° C and a standard deviation of 1.00°. Find the indicated probability, where \( z \) is the reading in degrees.

269/46 \( P(z < 1.645) \)

Finding Probability. In Exercises 45–48, assume that the readings on the thermometers are normally distributed with a mean of 0° C and a standard deviation of 1.00°. Find the indicated probability, where \( z \) is the reading in degrees.

269/48 \( P(z < -1.96 \text{ or } z > 1.96) \)

Finding Temperature Values. In Exercises 49–52, assume that thermometer readings are normally distributed with a mean of 0° C and a standard deviation of 1.00° C. A thermometer is randomly selected and tested. In each case, draw a sketch, and find the temperature reading corresponding to the given information.

269/50 Find \( P_1 \), the 1st percentile. This is the temperature reading separating the bottom 1% from the top 99%.

Finding Temperature Values. In Exercises 49–52, assume that thermometer readings are normally distributed with a mean of 0° C and a standard deviation of 1.00° C. A thermometer is randomly selected and tested. In each case, draw a sketch, and find the temperature reading corresponding to the given information.

269/52 If 0.5% of the thermometers are rejected because they have readings that are too low and another 0.5% are rejected because they have readings that are too high, find the two readings that are cutoff values separating the rejected thermometers from the others.